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DIRECT TESTIMONY

OF

JOHN H. O'BANNON

ON BEHALF OF

THE OFFICE OF THE CONSUMER ADVOCATE

DECEMBER 30, 1997

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JOHN H. O'BANNON

STATEMENT OF QUALIFICATIONS

My name is John H. O'Bannon. I am currently a student in the doctoral program in the Department of Economics at the University of Virginia. I was awarded the Master of Arts in Economics from the University of Virginia in January of 1997. I received my Bachelor of Arts degree in May of 1995 from the College of William and Mary. My graduate focus involves Industrial Organization and Public Policy analysis.

I. PURPOSE AND SCOPE OF TESTIMONY

Upon close inspection, the testimony presented by Witness Mayes (USPS-T-37) regarding the Parcel Post category of mail, particularly concerning volume changes in particular rate cells in the test year that would prevail after the requested rate change, is theoretically perplexing. In particular, for some subsets of Parcel Post mail Witness Mayes' estimated volume changes in certain rate cells imply positive implicit own-price elasticities. This computational result challenges simple and universally accepted economic theory. Under typical assumptions (many of which the Postal Service itself invokes) positive implicit own-price elasticities are a theoretical and empirical impossibility. The Postal Service's current method of allocation of volume estimates to different rate cells within a category of mail is causing this problem.

II. POSITIVE IMPLICIT OWN-PRICE ELASTICITIES

In order to conduct my analysis, it was necessary to utilize the data presented in Witness Mayes' Workpapers (H197). In particular, I have used her Inter-BMC, Intra-BMC and DBMC information for four variables: TYBR volume, TYAR volume, R94-1 Rates, and her computed Phase Four (Final) Rates for these categories.[1] There are three subsets of Parcel Post that would exist unchanged both before and after the imposition of R97-1 rates. Using this data I computed the resulting own-price elasticities using a constant elasticity formula.

Own-Price Elasticity Formula:

$$\frac{\textit{Volume Before Rate Change}\,(v1)}{\textit{Volume After Rate Change}\,(v2)} = \left(\frac{\textit{Rate Before}\,(p1)}{\textit{Rate After}\,(p2)}\right)^{\varepsilon}$$

Solving for the value of the implicit own-price elasticity (ε) yields:

$$\varepsilon = \frac{\ln\left(v1/v2\right)}{\ln\left(p1/p2\right)}.$$

Notice that for any cell in which the rate is unchanged, the implicit own-price elasticity will be undefined due to division by zero.

The result of performing this calculation on every rate cell in each of the three categories of Parcel Post mail service is presented in Appendix 3 following the text. What is immediately striking is that only for the DBMC category does one see

[1] TYBR volumes can be found in WP I.A. on pages 8-13. R94-1 Rates can be found in WP I.C. on pages 1-4 and 7-8. Phase Four (Final) Rates can be found in WP I.N. pages 1-6. TYAR volumes can be found in WP II.A. on pages 2-7.

positive elasticities. This results from the method by which the Postal Service distributes its volume change estimates across the rate cells.

A. Postal Service Volume Distribution Method.

It would be illustrative to briefly discuss how the Postal Service computes its volume estimates for each rate cell for any given category of mail.[2] The Postal Service knows the total volume for each category of mail for some historical period of four consecutive postal quarters. It then uses historical growth in volume data to estimate the total volume that would exist in the absence of a rate change. This total volume figure is then distributed across all the cells in each category in the exact proportion that existed during the historical year.

The Postal Service suggests new rates for each cell of service. It computes an overall rate weighted by the historical volumes to determine an overall rate for service. It uses this rate, in conjunction with the historical growth rate, to determine a new overall volume level for that category of service. At no time, however, does the Postal Service specifically examine the rate change in a particular cell and attempt to generate a volume estimate directly related to that individual cell's rate change.

[2] The method described is also the one employed by Witness Mayes in her Workpapers.

It is for this reason that one arrives at the economic anomaly that increasing the rate on a particular cell of service produces an increase in volume for that cell of service. This is true for almost all the cells in the DBMC category of service. This results from the fact that the Postal Service believes the overall volume will increase for DMBC despite the fact that all but two cells experience rate increases.[3]

[3] As presented in Witness Mayes' Workpapers, the TYAR DBMC volume is slightly less than TYBR volume when approximated in WP. I.A., p.1 However, TYAR DBMC volume exceeds TYBR volume in WP. II.A. p.1.

III. ECONOMIC IMPLAUSIBILITY OF POSITIVE OWN-PRICE ELASTICITIES

Simple economic theory can be used to show that positive own-price elasticities, under a general and widely accepted set of assumptions, while not impossible are highly improbable. Their improbability is such that even if in some cells a raise in price does bring about a rise in volume for that cell, overall the sum of price changes times corresponding volume changes must be negative (as described in Equation 7 below.) Using the data presented in Witness Mayes' Workpapers, one can test whether the Postal Service's volume estimates meet the stringent requirements for positive own-price elasticities to exist.

A. Non-compensated demand analysis

I will first prove the necessity of negative own-price elasticities using Marshallian demand analysis. First, assume there is some composite good that serves as "all other goods" in this analysis. Its price does not change, $p_{A0} = p_{B0}$. The prices of the Postal Service rate cells under investigation can rise or fall. Thus, using vector notation, $\bar{p}_A \neq \bar{p}_B$, where some price elements have risen, some may have fallen, and some may be unchanged.[4] The consumer's income, m, does not change. Thus, the consumer's total expenditure does not change after the price

[4] The vectors $\bar{p}_B$ and $\bar{p}_A$ are the vector of rates before and after the rate change, respectively. More explicitly $\bar{p}_A = (p_{A0}, p_{A1}, p_{A2}, \cdots, p_{AL})$ where L is the total number of goods.

change as one assumes the consumer spends all of his or her income to maximize utility.

I then use the Marshallian demand function. This function, $x_i(\bar{p},m)$ describes the quantity of good x_i the consumer chooses in order to maximize his or her utility when facing the price vector $\bar{p}$ and endowed with income m. The term $x(\bar{p}_B,m)$ is therefore the bundle of goods, the quantities of every particular Postal Service good's cell and the composite good, that the consumer has chosen in order to maximize his or her utility before the rate change.

The first basic assumption applied is that Postal Service goods are normal goods.[5] By the definition of a normal good we know:

$$\frac{\partial x_i(\bar{p},m)}{\partial m} \geq 0. \qquad \text{Assumption 1}$$

The next basic assumption applied is that each Postal Service good's cell within a category represents a good that is unrelated to every other cell in that category.[6] This implies:

$$\frac{\partial x_i(\bar{p},m)}{\partial p_j} = 0 \ \ \forall \ i \neq j. \qquad \text{Assumption 2}$$

One result, making direct use of the fact that the Marshallian demand function is homogenous of degree zero, that can be derived from Euler's formula[7] is:

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[5] This is certainly a restrictive assumption. In reality some cells of a particular category of parcel post may function as inferior goods. However, I do not believe that the Postal Service would argue this.

[6] Each cell is neither a substitute nor a complement for any other cell in that category.

$$\sum_{k=1}^{L} \frac{\partial x_i(\bar{p},m)}{\partial \bar{p}_k} p_k + \frac{\partial x_i(\bar{p},m)}{\partial m} m = 0 \text{ for } i = 1,\ldots,L. \qquad \text{Equation 1}$$

Making use of Assumption 2, the fact that each cell is unrelated to each other cell, the first term simplifies and one can state:

$$\frac{\partial x_i(\bar{p},m)}{\partial \bar{p}_i} p_i + \frac{\partial x_i(\bar{p},m)}{\partial m} m = 0. \qquad \text{Equation 2}$$

My assumptions state that each cell is a normal good, that income is positive, and that the rate for each cell is positive. Therefore, for the expression to equal zero given that $\frac{\partial x_i(\bar{p},m)}{\partial m} > 0 \Rightarrow \frac{\partial x_i(\bar{p},m)}{\partial m} m > 0$ if $m > 0$, the own-price term must be negative, and the resulting own-price elasticity would be negative. Thus, non-compensated demand analysis shows that positive own-price elasticities are theoretically impossible.

B. Compensated demand analysis

The use of Hicksian, or compensated demand analysis, allows one to examine the reactions of the consumer given that his or her utility remains constant. This is in contrast to the Marshallian analysis presented above, which holds the consumers' income constant and allows him or her to maximize utility at some other level. Thus, the proper application of Hicksian analysis requires one to always

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[7] This result is proved in Appendix 1.

compensate the consumer, by giving him or her a quantity of income, Δm, such that the original level of utility is still attainable under the new prices.[8]

I will now show the price change using vector notation in the following way:

$$\bar{p}_A = \bar{p}_B + \Delta\bar{p}$$

In this expression, $\Delta\bar{p}$ is a vector of the magnitudes of the price changes. A cell that has its price increased will be represented in $\Delta\bar{p}$ by a positive number, while a cell that has its price decreased will be represented in $\Delta\bar{p}$ by a negative number.

My first assertion is that the bundle $x(\bar{p}_B + \Delta\bar{p}, m + \Delta m)$ is viewed with indifference by the consumer to his or her original bundle $x(\bar{p}_B, m)$. Since neither bundle can be strictly revealed preferred, using a simple analysis of preferences we can say:

$$\bar{p}_B x(\bar{p}_B, m) \le \bar{p}_B x(\bar{p}_B + \Delta\bar{p}, m + \Delta m) \qquad \text{Equation 3}$$

$$(\bar{p}_B + \Delta\bar{p}) x(\bar{p}_B + \Delta\bar{p}, m + \Delta m) \le (\bar{p}_B + \Delta p) x(\bar{p}_B, m) \quad \text{Equation 4}$$

A two-goods diagram is used to derive these two equations in Appendix 2. Figure 1 shows that as the price of one of good changes the consumer's income is changed in such a way that he or she remains on the original indifference curve. Equations 3 and 4 can then be determined from points on the original indifference curve and points elsewhere on the two budget lines.

Summing the two inequalities from Equations 3 and 4 yields:

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[8] Here the use of the term price is interchangeable with the term postal rates.

$$\Delta\bar{p}\left[x(\bar{p}_B + \Delta\bar{p}, m + \Delta m) - x(\bar{p}_B, m)\right] \leq 0 . \qquad \text{Equation 5}$$

Rewriting the term inside the brackets as Δx, then the expression simplifies to:

$$\Delta\bar{p}\Delta x \leq 0 . \qquad \text{Equation 6}$$

Taking this out of vector form:

$$\sum_{i=1}^{L} \Delta p_i \Delta x_i \leq 0 . \qquad \text{Equation 7}$$

Next I can separate the cells by their price changes and the resulting changes in volume. Assume that I group all the cells for which the price has fallen into the first n of the L possible cells. All of these cells will experience an increase in volume. Cells $n+1$ through $n+j$ will be cells for which the price has risen, and the resulting volume change is negative. Cells $n+j+1$ through k will be cells for which the price has risen and the resulting volume change was positive. This is the type of cell that will generate positive own-price elasticities. Finally, cells $k+1$ through L are cells for which there was no change in price.

Thus the expression from Equation 7 can be rewritten as:

$$\sum_{i=1}^{n} \Delta p_i \Delta x_i + \sum_{i=n+1}^{n+j} \Delta p_i \Delta x_i + \sum_{i=n+j+1}^{k} \Delta p_i \Delta x_i + \sum_{i=k+1}^{L} \Delta p_i \Delta x_i \leq 0 \qquad \text{Equation 8}$$

The first term in Equation 8 is strictly negative, since volume increases from the price decreases. The last term in Equation 8 is zero, as I have not changed the prices of these cells and their resulting volume change is immaterial. The second term in Equation 8 is also strictly negative, as this term fits the standard economic implications that an increase in price brings about a decrease in consumption. The third term will be positive under the Postal Service's assumption that positive own-price elasticities exist.

Thus if the Postal Service's assertion is true, then the following regularity must hold in the data:

$$\left|\sum_{i=1}^{n+j} \Delta p_i \Delta x_i\right| \geq \sum_{i=n+j+1}^{k} \Delta p_i \Delta x_i \,. \qquad \text{Equation 9}$$

Equation 9 simply states that the magnitude of the sum of the product of the change in price with the change in consumption for cells that show an inverse relationship between the two variables must exceed the magnitude of this product for cells that show a direct relationship between these two variables. This is certainly a restrictive requirement that may or may not be supported by any particular data set. Hicksian analysis shows that the assertion of positive own-price elasticities, while not theoretically impossible, is highly restrictive.

IV. EMPIRICAL ANALYSIS

The result in Equation 9 shows that empirical analysis can be used, with the Postal Service's data, to determine if their tacit acceptance of positive own-price elasticities is supportable. From Equation 7, one sees that a simple calculation can be undertaken to test whether the Postal Service's use of positive own-price elasticities is supportable. If one multiplies each cell's price change with its expected volume change, and sums these values across all the cells in a given category of Parcel Post, then one should find the resulting quantity to be weakly negative.[9]

I computed the SMD values implied by Equation 7 and described immediately above for the Intra-BMC, Inter-BMC, and DBMC categories of Parcel Post.[10] It should be noted again that only the DBMC category revealed positive own-price elasticities, and thus it was the only category that I am testing empirically against the prior theoretical assumption implied by Equation 7. In line with expectations resulting from the theoretical results, the computed SMD values for the Intra-BMC and Inter-BMC categories were negative.[11] This agrees with the empirical fact that

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[9] The value resulting from the computation suggested by Equation 7 is hereafter referred to as the sum of multiplied differences (SMD).

[10] Tables showing the multiplied differences for each rate cell, and the sum of multiplied differences for each category are presented in Appendix 4 following the text.

[11] For Intra-BMC this value was -2,406,031. For Inter-BMC this value was -14,084,407.

neither of their own-price elasticities were positive. However, when I performed the calculation on the DBMC category of Parcel Post the resulting SMD quantity was positive.[12] This result does not imply that positive own-price elasticities cannot occur for cells within categories of Parcel Post. It only implies that the positive own-price elasticities derived in the case of DBMC Parcel Post contradict economic theory as revealed in the accompanying data.

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[12] The value was 4,303,124.

V. CONCLUSION

Through the examinations of simple economic theories it is clear that when considered theoretically, positive own-price elasticities are almost impossible. Compensated (Hicksian) analysis has been shown to allow positive own-price elasticities to exist. However, with the categories of Parcel Post under consideration, the empirical result that must be present in the data is highly restrictive. When this restriction is explored empirically in the data used by the Postal Service, and by Witness Mayes in particular, the result tends to discourage the possibility of positive own-price elasticities.

This result does not imply that positive own-price elasticities cannot occur for cells within categories of Parcel Post. It only implies that the particular positive own-price elasticities utilized in the case of DBMC Parcel Post are not theoretically supportable by the accompanying data. This means that some step in the Postal Service's process of allocating volume estimates to rate cells is flawed. A better system of estimating the volume resulting in each cell from that particular cell's rate change needs to be found.

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Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green, Microeconomic Theory, New York, Oxford University Press, 1995.

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This appendix includes the derivation of Equation 1 using Euler's formula and the fact that Marshallian demand is homogenous of degree zero.

Definition of Homogeneity of degree r

If we say the function $f(\bar{x})$ is homogenous of degree r, where $\bar{x} = (x_1, x_2, \cdots, x_l)$, then:

$$f(tx_1, tx_2, \cdots, tx_l) = t^r f(x_1, x_2, \cdots, x_l). \qquad \text{Appendix Equation 1}$$

Thus saying the Marshallian demand function is homogenous of degree zero means that $x(t\bar{p}, tm) = t^0 x(\bar{p}, m) = x(\bar{p}, m)$. That is, if prices and income rise by the same proportion (X%), then the quantities in the consumer's utility maximizing bundle are unchanged.

Euler's formula

Suppose that the function $f(x_1, x_2, \cdots x_l)$ is homogenous of degree r and once differentiable. Then at any $\bar{x}$, where $\bar{x} = (x_1, x_2, \cdots, x_l)$, we have:

$$\sum_{n=1}^{N} \frac{\partial f(\bar{x})}{\partial x_n} x_n = rf(\bar{x}). \qquad \text{Appendix Equation 2}$$

Proof of Euler's formula

Differentiate each side of Appendix Equation 1, from the definition of a homogenous function, with respect to t.

$$\frac{\partial}{\partial t} f(tx_1, tx_2, \cdots, tx_l) = \frac{\partial f(t\bar{x})}{\partial x_1} x_1 + \frac{\partial f(t\bar{x})}{\partial x_2} x_2 + \cdots + \frac{\partial f(t\bar{x})}{\partial x_l} x_l. \qquad \text{Appendix Equation 3}$$

We simplify the left-hand side of this equation by directly applying the definition of homogeneity:

$$\frac{\partial}{\partial t} f(tx_1, tx_2, \cdots, tx_l) = \frac{\partial}{\partial t}\left[t^r f(x_1, x_2, \cdots, x_l)\right]. \qquad \text{Appendix Equation 4}$$

Compute the derivative of the right-hand side of this expression with respect to t:

$$\frac{\partial}{\partial t}\left[t^r f(x_1, x_2, \cdots, x_l)\right] = rt^{r-1} f(x_1, x_2, \cdots, x_l). \qquad \text{Appendix Equation 5}$$

Now we set our simplified right-hand side from Appendix Equation 5 equal to the right-hand side from Appendix Equation 3:

$$rt^{r-1} f(x_1, x_2, \cdots, x_l) = \frac{\partial f(t\bar{x})}{\partial x_1} x_1 + \frac{\partial f(t\bar{x})}{\partial x_2} x_2 + \cdots + \frac{\partial f(t\bar{x})}{\partial x_l} x_l. \qquad \text{Appendix Equation 6}$$

We want to see how the function relates to itself identically, instead of how it relates to a proportional value of itself. For this reason we set $t = 1$ and find:

$$rf(x_1, x_2, \cdots, x_l) = \frac{\partial f(\bar{x})}{\partial x_1} x_1 + \frac{\partial f(\bar{x})}{\partial x_2} x_2 + \cdots + \frac{\partial f(\bar{x})}{\partial x_l} x_l. \qquad \text{Appendix Equation 7}$$

This can be rewritten as:

$$rf(\bar{x}) = \sum_{n=1}^{N} \frac{\partial f(\bar{x})}{\partial x_n} x_n \qquad \text{Appendix Equation 8}$$

Notice that Appendix Equation 8 is identical to Appendix Equation 2. Thus we have proven Euler's formula.

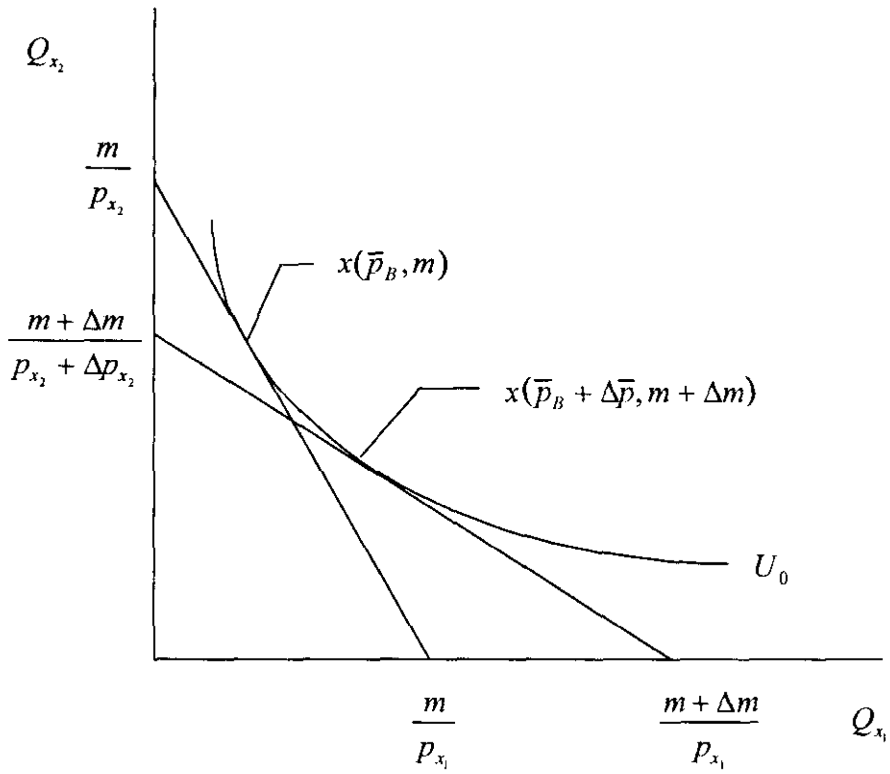
Next we substitute the Marshallian demand function for the function $f(\bar{x})$ in Appendix Equation 7, such that $f(\bar{x}) = x_i(\bar{p}, m)$ for each $i = 1, \cdots, L$. We also make use of the fact that this function is homogenous of degree zero, such that $r = 0$. Appendix Equation 8 is now:

$$0 = \sum_{k=1}^{L} \frac{\partial x_i(\bar{p}, m)}{\partial p_k} + \frac{\partial x_i(\bar{p}, m)}{\partial m} \text{ for } i = 1, \cdots L \qquad \text{Appendix Equation 9}$$

Appendix Equation 9 is identical to Equation 1 used in the body of the text.

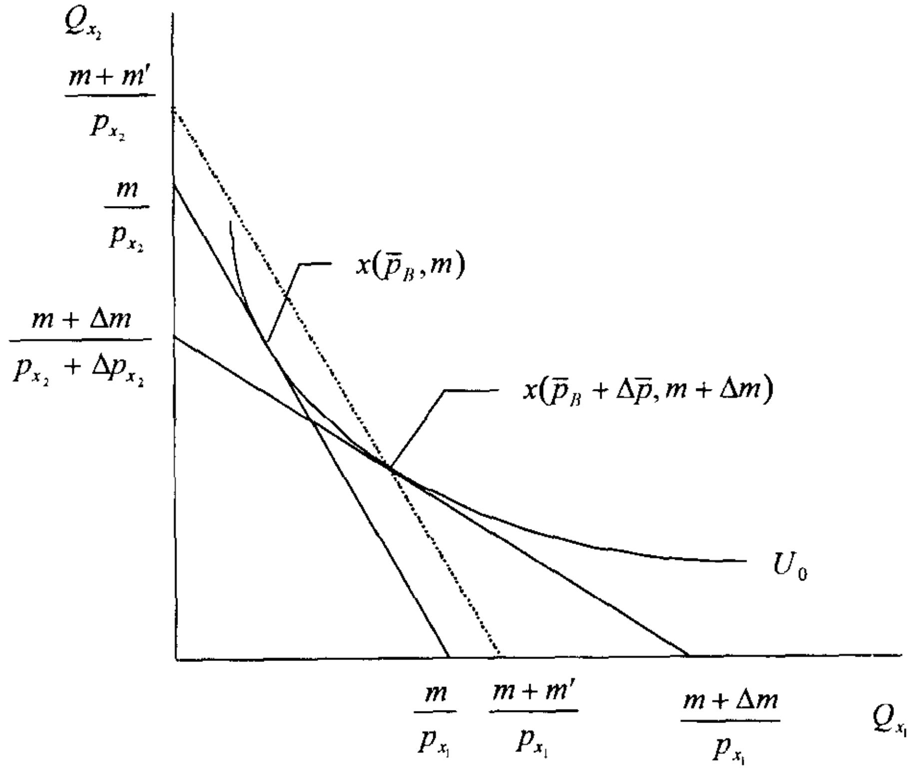
This appendix includes the theoretical underpinnings for Equations 3 and 4 presented in the text.

Figure 1



The above two-good graph depicts the situation described in the text dealing with the Compensated demand analysis. The bundle $x(\bar{p}_B, m)$ is the utility maximizing quantities of the two goods that the consumer chooses under the initial set of prices, $\bar{p}_B$, and his or her initial level of income, m. Hicksian analysis begins by describing the bundle $x(\bar{p}_B+\Delta\bar{p}, m+\Delta m)$. This is the bundle that the consumer would choose after the price change in the good x_2, from p_{x_2} to $p_{x_2} + \Delta p_{x_2}$, while simultaneously being given Δm such that he or she can exactly attain the original level of utility U_0.

From this graph we can derive Equations 3 and 4 from the text. Consider if the consumer attempted to purchase the second bundle $x(\bar{p}_B + \Delta\bar{p}, m + \Delta m)$ at the original prices $\bar{p}_B$. He or she would find this bundle unaffordable given the original income m as depicted in Figure 1. In general, for well-behaved preferences, the new bundle will be more costly than the original bundle at the original prices. The consumer would have needed additional income, m', in order to purchase the new bundle at the original prices. This is shown in Figure 2 below.

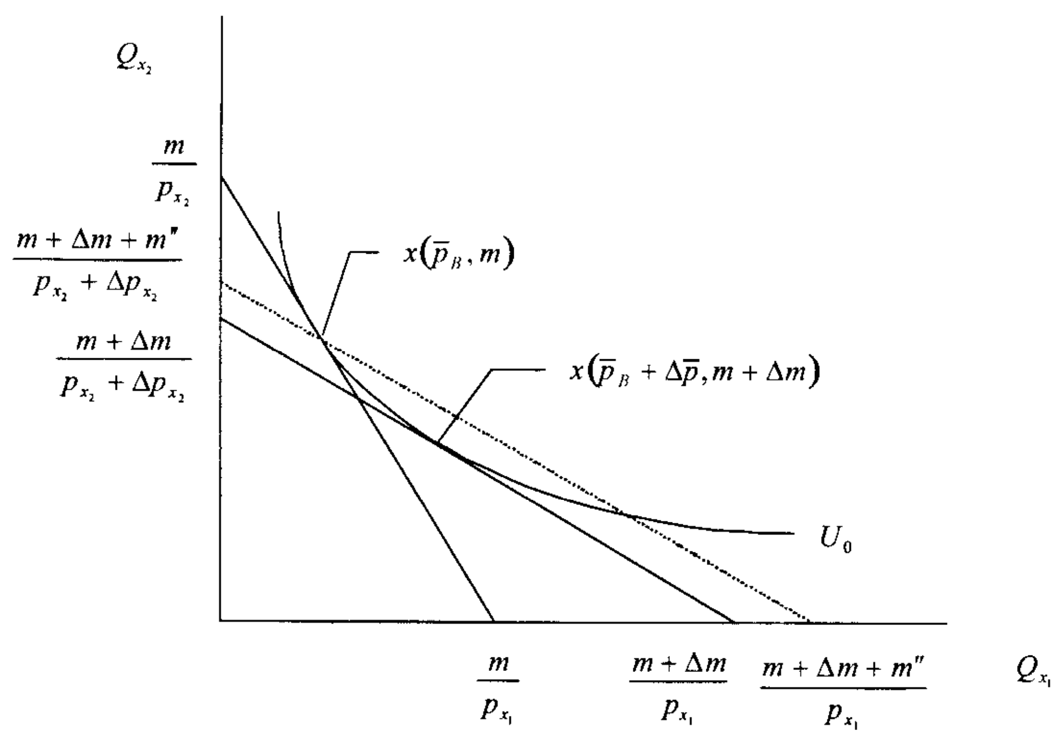


Thus Figure 2 graphically depicts the situation described by Equation 3 in the text. Specifically,

$$\bar{p}_B x(\bar{p}_B, m) \leq \bar{p}_B x(\bar{p}_B + \Delta\bar{p}, m + \Delta m). \qquad \text{Equation 3}$$

Now consider if the consumer attempted to purchase the original bundle $x(\bar{p}_B, m)$ at the new prices $\bar{p}_B + \Delta\bar{p}$. Again as depicted in Figure 1, he or she would find this bundle unaffordable given income $m + \Delta m$, which is the amount required to purchase the new bundle at the new prices. For well-behaved preferences, the old bundle will be more costly than the new bundle at the new prices. The consumer would need some additional income, m'', beyond the amount $m + \Delta m$, in order to be able to afford the old bundle at the new prices. This is depicted in Figure 3 below.

<u>Figure 3</u>



Thus Figure 3 graphically depicts the situation described by Equation 4 in the text. Specifically,

$$(\bar{p}_B + \Delta\bar{p})x(\bar{p}_B + \Delta\bar{p}, m + \Delta m) \leq (\bar{p}_B + \Delta p)x(\bar{p}_B, m). \text{ Equation 4}$$

This concludes the derivation of Equations 3 and 4 from the text.